Wealth distribution on a dynamic complex network

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Although an old problem, income and wealth disparities have increased substantially since the early XXI century [1, 2]. Despite the geographical, cultural, and historical differences, income distribution in different countries follows the same pattern [3], and understanding the mechanisms responsible for the origin and growth of inequalities is crucial to avoid it. Economic models based on statistical physics methods proposed in recent years [4] try to shed some light on this problem. In particular, agent-based models, where an agent is characterized by its wealth ω_i and savings fraction α_i. During the dynamics of the model, the agents exchange wealth, interacting in pairs following certain rules. One of the main strengths of this class of models is that different factors can be easily incorporated and studied, such as different rules for taxes and wealth exchanges [5].

On the other hand, a growing and promising field of study in economics is complex network theory [6]. In this approach, financial institutions or economic agents are the network's nodes, and relations among them are the edges. However, few contributions include complex networks into agent-based economic models [7, 8], and even in those, the network topology is fixed in time. Our goal is to provide a framework where a dynamic complex network is incorporated into an agent-based model, enabling a topological analysis of economic inequality phenomena. To study the disparities arising from economic transactions, we consider a conservative market, thus excluding the influences of processes such as the production of wealth and capital appreciation.

To create a dynamic network related to the wealth exchange process, we propose a model that considers that an agent's degree depends on its wealth. We justify this idea by considering that wealthier financial institutions can create more diversified investment portfolios. In the same way, wealthier companies or industries can reach a more significant number of investors or consumers. Thus, we propose a model which alternates between two dependent processes: the exchange of wealth between connected agents and the rewiring of the network connections. For the exchange of wealth we use the the yard-sale model, where the wealth exchanged between agents i and j is the minimum that the agents put at stake, $dw = \min[\alpha_i\omega_i, \alpha_j\omega_j]$. The wealth of agents i and j after the exchange is $w_i(t+1) = w_i(t) + dw$, and $w_j(t+1) = w_j(t) - dw$, so the total wealth is conserved. We assume a probability of the poorest agent winning the transaction given by

$$p_{i,j} = \frac{1}{2} + f \times \frac{|\omega_i(t) - \omega_j(t)|}{\omega_i(t) + \omega_j(t)}, \tag{1}$$

where f is the social protection factor, which varies from 0 to 1/2. The rewiring process starts by randomly selecting a pair i, j of agents, if this pair is disconnected the probability of creating a new connection follows

$$P_{i,j} = \frac{\omega_i(t) + \omega_j(t)}{\sum_l \omega_l(t)}, \tag{2}$$

where the sum in l is only on agents with at least one connection and $\omega_i(t)$ is the wealth of agent i at time t. If the pair is already connected, the link breaks with the complementary probability, $Q_{i,j} = 1 - P_{i,j}$. In order

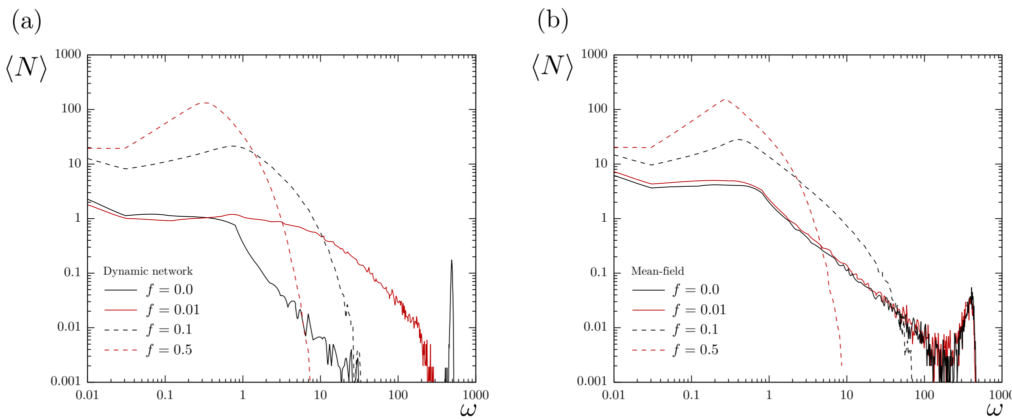

Figure 1: Non-accumulated distribution of wealth for different values of social protection in the (a) dynamic network and (b) mean-field model.

to all the agents being able to participate in this process, we randomly select $N/2$ pairs of agents to rewire their connections. After this process, we go back to the wealth exchange dynamics over the updated network structure. In this way, the system's evolution depends on wealth exchange and rewiring processes, which are dependent. The two process, wealth exchange between all the connected pairs, and the network rewiring of $N/2$ pairs, defines one Monte Carlo Step (MCS). We perform simulations for systems with size $N = 10^3$ over $t = 4 \cdot 10^4 MCS$. The results are averaged over 10^3 independent samples.

We obtain results for different values of f, analyzing economic and topological indicators, such as the Gini index, the distribution of wealth (Fig 1), assortativity (Fig 2), and the degree distribution. For the economic indicators, we compare our results with a mean-field model.

As shown in Fig 1, in the absence of social protection both models point out a condensation of wealth. Nevertheless, in the present model there is a gap in the wealth distribution, indicating a strong separation of classes, not present in the mean-field model results. The divergences between the two models are even more clear for $f = 0.01$, as the mean-field seems to be very robust to small increases of f, the dynamic network model presents a very distinct phenomenology from that of $f = 0.0$. As the social protection factor increases, the distributions of wealth for both models approach each other, yet the dynamic network model leads to smaller values of the Gini index. This results can be explained by the non-assortative behavior of the network as $f \to 0.5$, so the degree of the agents is not correlated with their neighbours (Fig 2).

In summary, our results showed that the dynamic network process has strong consequences in the agent based model, presenting a much richer phenomenology, especially in lower values of f. In our opinion, this simple model is able to reproduce distinct topological features which can be related to real societies, such as social stratification and the marginalization of the poorer agents.

Keywords: Econophysics, Wealth distribution, Economic networks, Agent-based models

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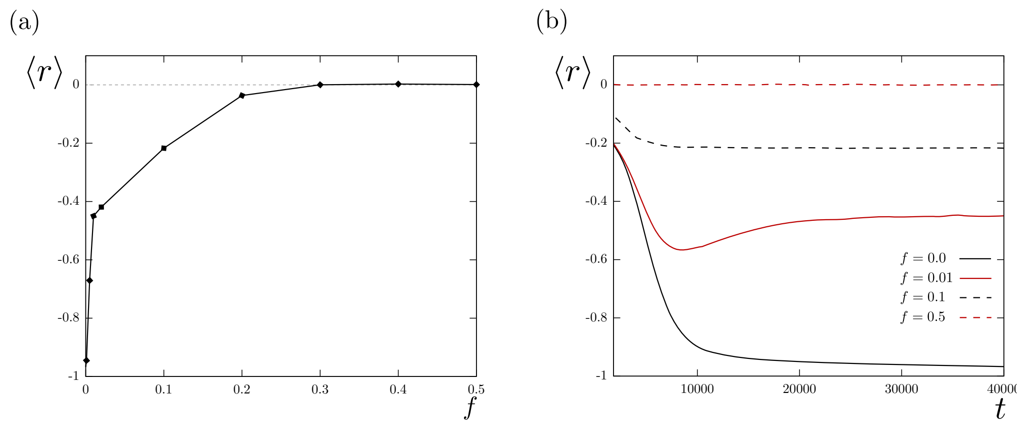

Figure 2: Assortativity of the network as a function of (a) social protection f and (b) time t for different values of f.

References

[1] L. Chancel, T. Piketty, Global Income Inequality, 1820–2020: the Persistence and Mutation of Extreme Inequality, Journal of the European Economic Association 19 (6) (2021) 3025–3062. doi:10.1093/jeea/jvab047.

[2] E. Saez, G. Zucman, Wealth inequality in the united states since 1913: Evidence from capitalized income tax data, The Quarterly Journal of Economics 131 (2) (2016) 519–578.

[3] B. K. Chakrabarti, A. Chakraborti, S. R. Chakravarty, A. Chatterjee, Econophysics of income and wealth distributions (2013).

[4] V. M. Yakovenko, J. B. Rosser Jr, Colloquium: Statistical mechanics of money, wealth, and income, Reviews of modern physics 81 (4) (2009) 1703.

[5] J. R. Iglesias, B.-H. F. Cardoso, S. Gonçalves, Inequality, a scourge of the xxi century, Communications in Nonlinear Science and Numerical Simulation 95 (2021) 105646.

[6] M. Bardoscia, P. Barucca, S. Battiston, F. Caccioli, G. Cimini, D. Garlaschelli, F. Saracco, T. Squartini, G. Caldarelli, The physics of financial networks, Nature Reviews Physics (2021) 1–18.

[7] T. Ma, J. G. Holden, R. Serota, Distribution of wealth in a network model of the economy, Physica A: Statistical Mechanics and its Applications 392 (10) (2013) 2434–2441.

[8] L. A. Braunstein, P. A. Macri, J. R. Iglesias, Study of a market model with conservative exchanges on complex networks, Physica A: Statistical Mechanics and its Applications 392 (8) (2013) 1788–1794.